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Inventory Model for Imperfect Quality and Repairable items with Varying Deterioration

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ABSTRACT: Many times it happens that units produced or ordered are not of 100% good quality. Some of them may be repairable and some may not be repairable. A deterministic inventory model with variable deterioration rates for imperfect quality and repairable items is developed. Demand is considered as linear function of time. Shortages are not allowed. Numerical example is given to support the model. Sensitivity analysis is done for parameters.

Key Words: Inventory model, Varying Deterioration, Linear demand, Defective items, Repairable items

I. INTRODUCTION

Many researchers have discussed inventory models for deteriorating items in past. Within [17] discussed deteriorating items inventory model for fashion goods. Ghare and Schrader [3] considered inventory problem under constant demand and constant deterioration. Shah and Jaiswal [14] considered an order level inventory model for items deteriorating at a constant rate. An order level inventory model with constant rate of deterioration was studied by Aggarwal [1]. A deteriorating items inventory model with linear trend in demand was discussed by Dave and Patel [2]. Salameh and Jaber [13] developed a model to determine the total profit per unit of time and the economic order quantity for a product purchased from the supplier. Mukhopadhyay *et al.* [8] developed an inventory model for deteriorating items with a price-dependent demand rate. The rate of deterioration was taken to be time-proportional and a power law form of the price-dependence of demand was considered. Teng and Chang [15] considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Mathew [7] developed an inventory model for deteriorating items with mixture of Weibull rate of decay and demand as function of both selling price and time. Other research work related to deteriorating items can be found in, for instance (Raafat [11], Goyal and Giri [5], Ruxian *et al.* [12]).

Koh *et al.* [6] considered a joint EOQ and EPQ model in whch the stationary demand can be satisfied by recycled products and newly purchased products. Yadav and Kumar [18] developed an inventory model for repairable items with linear demand. Gothi *et al.* [4] considered an inventory model for repairable items with exponential deterioration and linear demand. Uthayakumar and Sekar [16] developed a multiple production setups inventory model for imperfect items with salvage value.

Generally the products are such that initially there is no deterioration. Deterioration starts after certain time and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model for imperfect quality and repairable items with different deterioration rates for the cycle time. Shortages are not allowed. Numerical example is provided to illustrate the model. Sensitivity analysis for major parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

Notations: The following notations are used for the development of the model:

$D(t)$: Demand rate is a linear function of time t ($a+bt$, $a>0$, $0<b<1$)
c : Purchasing cost per unit
p : Selling price per unit
d : defective items (%)
$1-d$: good items (%)
d_1 : repairable items (%)
λ : Screening rate
SR : Sales revenue
A : Replenishment cost per order for
z : Screening cost per unit
p_d : Price of defective items per unit
$h(t)$: Variable Holding cost $(x + yt)$
m : Transportation cost per unit of repairable items
t_1 : Screening time
T : Length of inventory cycle
$I(t)$: Inventory level at any instant of time t, $0 \le t \le T$
Q : Order quantity
θ : Deterioration rate during $\mu_1 \le t \le \mu_2$, $0< \theta<1$
θt : Deterioration rate during , $\mu_2 \le t \le T$, $0< \theta<1$
π : Total relevant profit per unit time.

Assumptions: The following assumptions are considered for the development of model.

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- The screening process and demand proceeds simultaneously but screening rate (λ) is greater than the demand rate i.e. $\lambda > (a+bt)$.
- The defective items and repairable items are independent of deterioration.
- Deteriorated units can neither be repaired nor replaced during the cycle time.
- A single product is considered.
- Holding cost is time dependent.
- The screening rate (λ) is sufficiently large. In general, this assumption should be acceptable since the automatic screening machine usually takes only little time to all items produced or purchased.

III. THE MATHEMATICAL MODEL AND ANALYSIS

In the following situation, Q items are received at the beginning of the period. Each lot having a d % defective items out of which d_1% are repairable items. The nature of the inventory level is shown in the given figure, where screening process is done for all the received items at the rate of λ units per unit time which is greater than demand rate for the time period 0 to t_1. During the screening process the demand occurs parallel to the screening process and is fulfilled from the goods which are found to be of perfect quality by screening process. The defective items are sold immediately after the screening process at time t_1 as a single batch at a discounted price and repairable items are sent back to manufacturer for repair. After the screening process at time t_1 the inventory level will be $I(t_1)$ and at time T, inventory level will become zero due to demand and partially due to deterioration.

Also here
$$t_1 = \frac{Q}{\lambda} \tag{1}$$
and defective percentage (d) is restricted to
$$d \le 1 - \frac{(a+bt)}{\lambda} \tag{2}$$

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by
$$\frac{dI(t)}{dt} = -(a+bt), \qquad 0 \le t \le \mu_1 \tag{3}$$
$$\frac{dI(t)}{dt} + \theta I(t) = -(a+bt), \qquad \mu_1 \le t \le \mu_2 \tag{4}$$

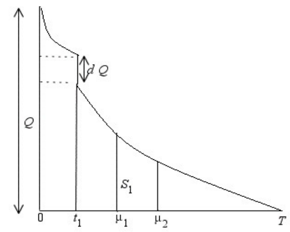


Fig. 1

$$\frac{dI(t)}{dt} + \theta I(t) = -(a+bt), \qquad \mu_2 \le t \le T \tag{5}$$
with initial conditions $I(0)=Q$, $I(\mu_1)=S_1$ and $I(T)=0$.

Solutions of these equations are given by
$$I(t) = Q - (at + \frac{1}{2}bt^2), \tag{6}$$
$$I(t) = \begin{bmatrix} a(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{2}a\theta(\mu_1^2 - t^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{bmatrix} + S_1\left[1 + \theta(\mu_1 - t)\right] \tag{7}$$
$$I(t) = \begin{bmatrix} a(T-t) + \frac{1}{2}b(T^2 - t^2) + \frac{1}{6}a\theta(T^3 - t^3) \\ + \frac{1}{8}b\theta(T^4 - t^4) - \frac{1}{2}a\theta t^2(T-t) \\ - \frac{1}{4}b\theta t^2(T^2 - t^2) \end{bmatrix}. \tag{8}$$
(by neglecting higher powers of θ)

After screening process, the number of defective items at time t_1 is dQ.

So effective inventory level during $t_1 \le t \le T$ is given by
$$I(t) = -(at + \frac{1}{2}bt^2) + Q(1-d). \tag{9}$$
From equation (6), putting $t = \mu_1$, we have
$$Q = S_1 + \left(a\mu_1 + \frac{1}{2}b\mu_1^2\right). \tag{10}$$
Moreover, we assume that d_1% of repairable items are sent to the manufacturer for repairs and after repairing become perfect or good items and received before the end of the cycle (i.e. during the time period $\mu_2 \le t \le T$). So it can be sold at original price before the end of the cycle. Doing so there is increase in transportation cost for sending and receiving back the defective repaired items. So effective inventory level during $\mu_2 \le t \le T$ is given by:

$$I(t)=\begin{bmatrix} a(T-t)+\frac{1}{2}b\left(T^2-t^2\right)+\frac{1}{6}a\theta\left(T^3-t^3\right) \\ +\frac{1}{8}b\theta\left(T^4-t^4\right)-\frac{1}{2}a\theta t^2(T-t) \\ -\frac{1}{4}b\theta t^2\left(T^2-t^2\right)+d_1Q \end{bmatrix}. \quad (11)$$

From equations (7) and (11), putting $t = \mu_2$, we have

$$I(\mu_2)=\begin{bmatrix} a(\mu_1-\mu_2)+\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ +\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)+\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right) \\ -a\theta\mu_2(\mu_1-\mu_2)-\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{bmatrix} + S_1\left[1+\theta(\mu_1-\mu_2)\right] \quad (12)$$

$$I(\mu_2)=\begin{bmatrix} a(T-\mu_2)+\frac{1}{2}b\left(T^2-\mu_2^2\right) \\ +\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \end{bmatrix} + d_1Q. \quad (13)$$

So from equations (12) and (13), we get

$$S_1=\frac{1}{\left[1+\theta(\mu_1-\mu_2)-d_1\right]}\begin{bmatrix} a(T-\mu_2)+\frac{1}{2}b\left(T^2-\mu_2^2\right) \\ +\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a(\mu_1-\mu_2)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right) \\ +a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \\ +d_1\left(a\mu_1+\frac{1}{2}b\mu_1^2\right) \end{bmatrix}. \quad (14)$$

Putting value of S_1 from equation (14) into equation (10), we have

$$Q=\frac{1}{\left[1+\theta(\mu_1-\mu_2)-d_1\right]}\begin{bmatrix} a(T-\mu_2)+\frac{1}{2}b\left(T^2-\mu_2^2\right) \\ +\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a(\mu_1-\mu_2)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right) \\ +a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \\ +d_1\left(a\mu_1+\frac{1}{2}b\mu_1^2\right) \end{bmatrix} + \left(a\mu_1+\frac{1}{2}b\mu_1^2\right). \quad (15)$$

Using (15) in (6), we have

$$I(t)=\frac{1}{\left[1+\theta(\mu_1-\mu_2)-d_1\right]}\begin{bmatrix} a(T-\mu_2)+\frac{1}{2}b\left(T^2-\mu_2^2\right) \\ +\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a(\mu_1-\mu_2)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right) \\ +a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \\ +d_1\left(a\mu_1+\frac{1}{2}b\mu_1^2\right) \end{bmatrix} + a(\mu_1-t)+\frac{1}{2}b\left(\mu_1^2-t^2\right). \quad (16)$$

Similarly, using (15) in (9), we have

$$I(t_1) = \frac{(1-d)}{\left[1+\theta(\mu_1-\mu_2)-d_1\right]} \begin{bmatrix} a(T-\mu_2) + \frac{1}{2}b(T^2-\mu_2^2) \\ +\frac{1}{6}a\theta(T^3-\mu_2^3)+\frac{1}{8}b\theta(T^4-\mu_2^4) \\ -\frac{1}{2}a\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2) \\ -a(\mu_1-\mu_2) - \frac{1}{2}b(\mu_1^2-\mu_2^2) \\ -\frac{1}{2}a\theta(\mu_1^2-\mu_2^2)-\frac{1}{3}b\theta(\mu_1^3-\mu_2^3) \\ +a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \\ +d_1\left(a\mu_1+\frac{1}{2}b\mu_1^2\right) \end{bmatrix} + (1-d)\left(a\mu_1+\frac{1}{2}b\mu_1^2\right) - (at+\frac{1}{2}bt^2) \quad (17)$$

Similarly putting value of S_1 from equation (14) in equation (7), we have

$$I(t) = \frac{\left[1+\theta(\mu_1-t)\right]}{\left[1+\theta(\mu_1-\mu_2)-d_1\right]} \begin{bmatrix} a(T-\mu_2)+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}a\theta(T^3-\mu_2^3) \\ +\frac{1}{8}b\theta(T^4-\mu_2^4)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2) \\ -a(\mu_1-\mu_2)-\frac{1}{2}b(\mu_1^2-\mu_2^2)-\frac{1}{2}a\theta(\mu_1^2-\mu_2^2) \\ -\frac{1}{3}b\theta(\mu_1^3-\mu_2^3)+a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \\ +d_1\left(a\mu_1+\frac{1}{2}b\mu_1^2\right) \end{bmatrix} + \begin{bmatrix} a(\mu_1-t)+\frac{1}{2}b(\mu_1^2-t^2)+\frac{1}{2}a\theta(\mu_1^2-t^2) \\ +\frac{1}{3}b\theta(\mu_1^3-t^3)-a\theta t(\mu_1-t)-\frac{1}{2}b\theta t(\mu_1^2-t^2) \end{bmatrix} \quad (18)$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements:

(i) Ordering cost (OC) = A (19)
(ii) Transportation cost (TC) = md_1Q (20)
(iii) Screening cost (SrC) = zQ (21)

(iv) $HC = \int_0^T (x+yt)I(t)dt$

$$= \int_0^{t_1}(x+yt)I(t)dt + \int_{t_1}^{\mu_1}(x+yt)I(t)dt + \int_{\mu_1}^{\mu_2}(x+yt)I(t)dt + \int_{\mu_2}^{T}(x+yt)I(t)dt \quad (22)$$

(v) $DC = c\left(\int_{\mu_1}^{\mu_2}\theta I(t)dt + \int_{\mu_2}^{T}\theta tI(t)dt\right)$ (23)

(vi) SR = Sum of sales revenue generated by demand meet during the period (0,T)
+ Sales of imperfect quality items
+ Sales of repaired items

$$= p\left(\int_0^T(a+bt)dt + p_d dQ + pd_1Q\right)$$

$$= p\left(aT + \frac{1}{2}bT^2 + p_d dQ + pd_1Q\right) \quad (24)$$

(by neglecting higher powers of θ)

The total profit (π) during a cycle consisted of the following:

$$\pi = \frac{1}{T}\left[SR - OC - SrC\ TC - HC - DC\right] \quad (25)$$

Substituting values from equations (19) to (24) in equation (25), we get total profit per unit. Putting $\mu_1 = v_1T$ and $\mu_2 = v_2T$ and value of t_1 and Q in equation (25), we get profit in terms of T. Differentiating equation (25) with respect to T and equate it to zero, we have

i.e. $\frac{d\pi}{dT} = 0$ (26)

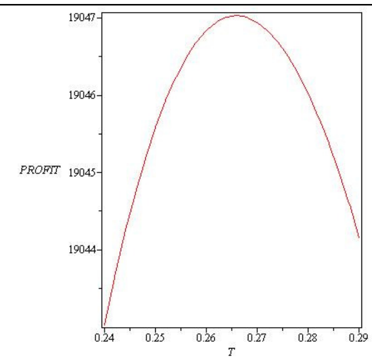
provided it satisfies the condition

$$\frac{d^2\pi}{dT^2} < 0. \quad (27)$$

IV. NUMERICAL EXAMPLE

Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, p_d = 15, d= 0.05, z = 0.40, d_1= 0.05, m = Rs. 50, λ= 10000, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, in appropriate units. The optimal value of T* =0.2659, Profit*= Rs. 19047.0329 and optimum order quantity Q* = 137.7645.
The second order conditions given in equation (27) are also satisfied. The graphical representation of the concavity of the profit function is also given.

T and Profit



Graph 1

IV. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1: Sensitivity Analysis.

Para-meter	%	T	Profit	Q
a	+20%	0.2429	22934.5963	150.9469
	+10%	0.2536	20990.0167	144.4946
	-10%	0.2801	17105.8926	130.6477
	-20%	0.2969	15166.9147	123.1396
θ	+20%	0.2637	19041.8042	136.7574
	+10%	0.2648	19044.4126	137.2615
	-10%	0.2670	19049.6651	138.2665
	-20%	0.2681	19052.3096	138.7673
x	+20%	0.2446	18980.9066	126.6740
	+10%	0.2546	19013.2798	131.8796
	-10%	0.2788	19082.3564	144.4863
	-20%	0.2938	19119.4887	152.3070
A	+20%	0.2909	18975.1927	150.7946
	+10%	0.2786	19010.3076	144.3820
	-10%	0.2524	19085.6170	130.7341
	-20%	0.2382	19126.3801	123.3436
λ	+20%	0.2660	19047.3285	137.8166
	+10%	0.2659	19047.1941	137.7645
	-10%	0.2658	19046.8358	137.7125
	-20%	0.2657	19046.5896	137.6604

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of θ and x, there is corresponding decrease/ increase in total profit and optimum order quantity.

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

For parameter λ, there is almost no change in profit.

VI. PARTICULAR CASE

(i) When d=0.02, m=0, d_1=0, we get
T^* = 0.2724, Profit = Rs. 19220.7443,
. which is same as Naik and Patel [9]

(ii) When pd=0, d = 0, m =0, d_1=0, we get
T^* = 0.2712, Profit = Rs. 19267.4380,
which is same as Patel and Sheikh [10].

VII. CONCLUSION

In this paper, we have developed an inventory model for defective and repairable deteriorating items with linear demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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